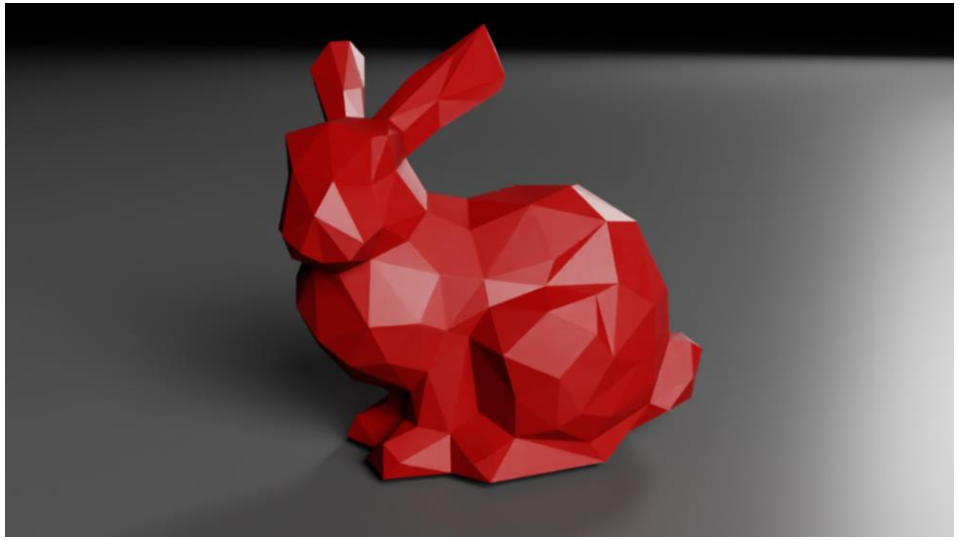
Unbreakable Soft Body Simulation with XPBD

Matthias Müller, Ten Minute Physics

www.matthiasmueller.info/tenMinutePhysics



Encounters with Soft Body Equations

$$\frac{\partial \mathbf{P}}{\partial f_{ij}} = \mu \frac{\partial \mathbf{F}}{\partial f_{ij}} + \frac{\mu}{J^2} \frac{\partial J}{\partial f_{ij}} \frac{\partial J}{\partial \mathbf{F}} - \frac{\mu}{J} \frac{\partial^2 J}{\partial \mathbf{F} \partial f_{ij}} + \frac{\lambda \log J}{J} \frac{\partial^2 J}{\partial \mathbf{F} \partial f_{ij}} - \frac{\lambda \log J}{J^2} \frac{\partial J}{\partial f_{ij}} \frac{\partial J}{\partial \mathbf{F}} + \frac{\lambda}{J^2} \frac{\partial J}{\partial f_{ij}} \frac{\partial J}{\partial \mathbf{F}}.$$

$$\mathbb{G}_J = \begin{bmatrix} \left[\frac{\partial J}{\partial f_{00}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{01}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{02}} \frac{\partial J}{\partial \mathbf{F}}\right] \\ \left[\frac{\partial J}{\partial f_{10}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{11}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{12}} \frac{\partial J}{\partial \mathbf{F}}\right] \\ \left[\frac{\partial J}{\partial f_{20}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{21}} \frac{\partial J}{\partial \mathbf{F}}\right] & \left[\frac{\partial J}{\partial f_{22}} \frac{\partial J}{\partial \mathbf{F}}\right] \end{bmatrix}$$

$$\frac{\partial \mathbf{P}}{\partial f_{ij}} = \mu \frac{\partial \mathbf{F}}{\partial f_{ij}} + \left[\frac{\mu + \lambda(1 - \log J)}{J^2}\right] \frac{\partial J}{\partial f_{ij}} \frac{\partial J}{\partial \mathbf{F}} + \left[\frac{\lambda \log J - \mu}{J}\right] \frac{\partial^2 J}{\partial \mathbf{F} \partial f_{ij}}.$$

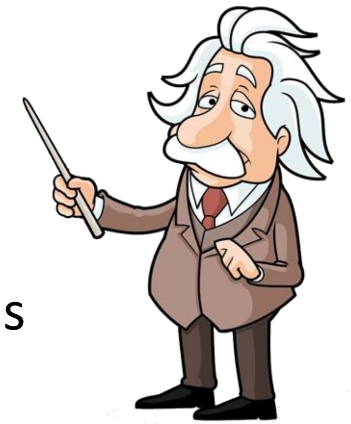
$$\begin{aligned} \operatorname{vec}\left(\frac{\partial^2 \Psi}{\partial \mathbf{F}^2}\right) = & \frac{\partial^2 \Psi}{\partial I_{\mathbf{C}}^2} \mathbf{g}_I \mathbf{g}_I^T + \frac{\partial \Psi}{\partial I_{\mathbf{C}}} \mathbf{H}_I + \\ & \frac{\partial^2 \Psi}{\partial II_{\mathbf{C}}^2} \mathbf{g}_{II} \mathbf{g}_{II}^T + \frac{\partial \Psi}{\partial II_{\mathbf{C}}} \mathbf{H}_{II} + \\ & \frac{\partial^2 \Psi}{\partial III_{\mathbf{C}}^2} \mathbf{g}_{III} \mathbf{g}_{III}^T + \frac{\partial \Psi}{\partial III_{\mathbf{C}}} \mathbf{H}_{III} \end{aligned}$$

$$\begin{aligned} \Psi_{\text{StVK, stretch}} &= \frac{1}{4} \|\mathbf{F}^T \mathbf{F} - \mathbf{I}\|_F^2 \\ &= \frac{1}{4} \left(\|\mathbf{F}^T \mathbf{F}\|_F^2 + \operatorname{tr} \mathbf{I} - 2 \operatorname{tr}(\mathbf{F}^T \mathbf{F}) \right) \\ &= \frac{1}{4} \left(\|\mathbf{F}^T \mathbf{F}\|_F^2 + \operatorname{tr} \mathbf{I} - 2 \|\mathbf{F}\|_F^2 \right). \end{aligned}$$

Dynamic Deformables, Theodore Kim and David Eberle, Siggraph Course 2020

Can we make things simpler?

Continuous Model & Global Solver



- Deformation as continuous vector functions
- Only possible in small regions
  → Finite Element Method (FEM)
- Mapping material behavior to functions of deformation functions
- Using concepts like 3rd order tensor derivatives
- The only possibilty before computers existed

Discrete model & Local solver

- Particles connected via simple constraints
  → like nature!
- Handling constraints one by one
- Using XPBD
  → Trivial implementation

Challenges of Traditional Simulations

Continuum models

- Recovery from Inversions
- Large rotations
- Volume conservation
- Large stretch / compression
- Finding constitutive models
- Strain limits
- High damping
- Complex equations

Global Solvers

- Linearization of equations
- Storing large matrices
- Handling non-symmetric matrices
- Under or over constrained systems
- Stability (line searchers)
- Handling inequalities (complementarity)
- Round friction cones

Challenges of Discrete Model and XPBD

None of the above

Discrete Model

- Tessellation has stronger effect on behavior

→

- Use uniform meshes
- Leverage to simulate anisotropic non-continuous detail
- Derive particle constraint from continuum model (upcoming tutorial)

Local Solver

- Convergence more slowly

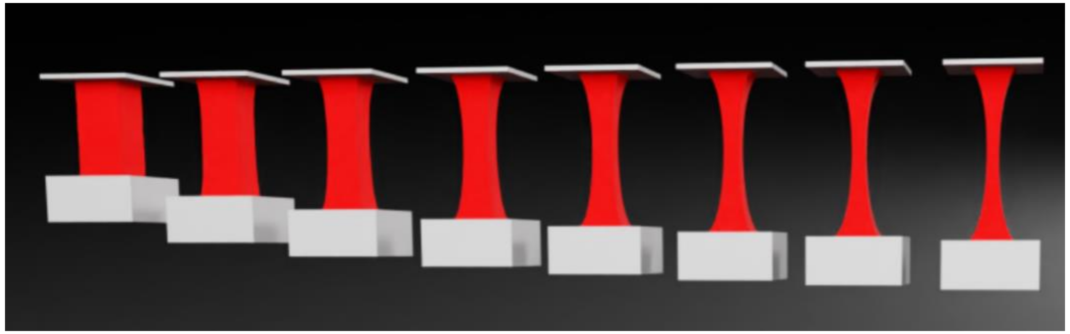
→

- Use sub-stepping

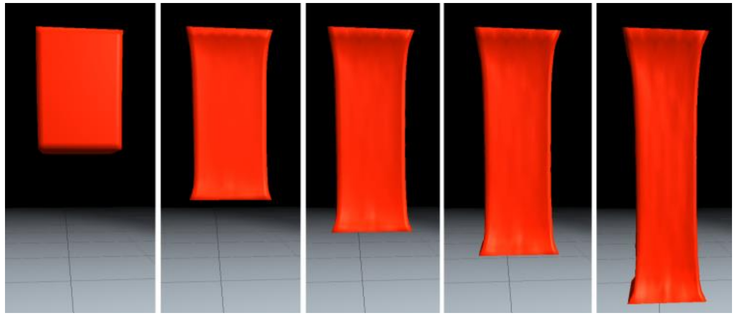
Accuracy

Accuracy

Ground truth



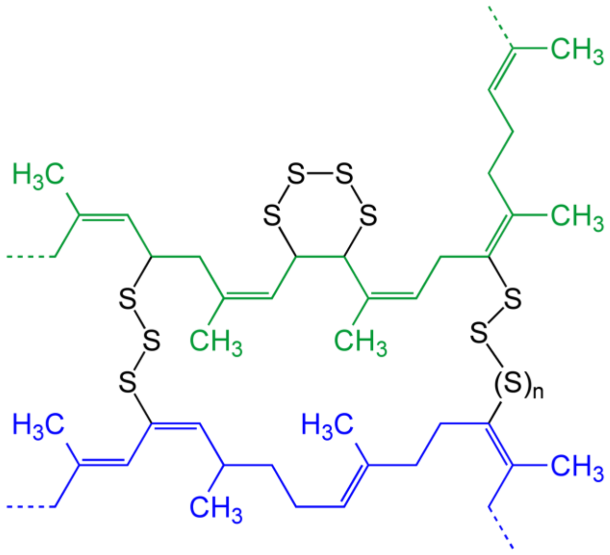
Hookean Model (popular in graphics)

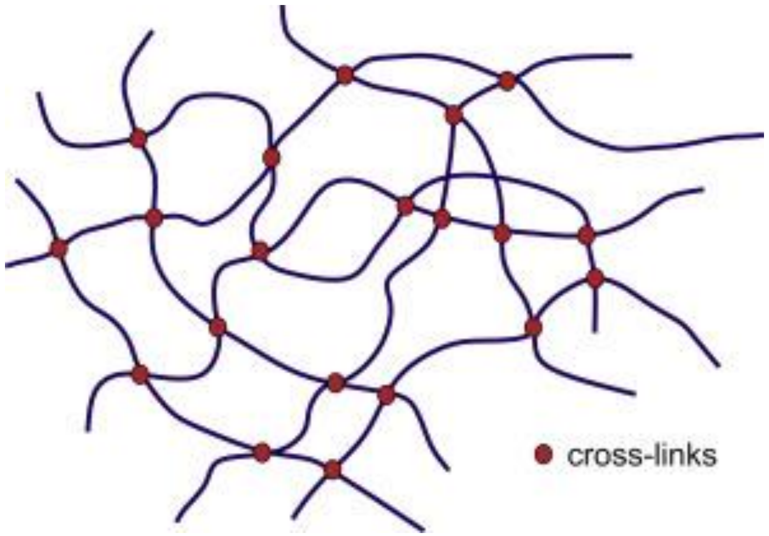


Discrete XPBD (browser demo)

Rubber – a Mass-Spring System

Polymer chains with cross links:



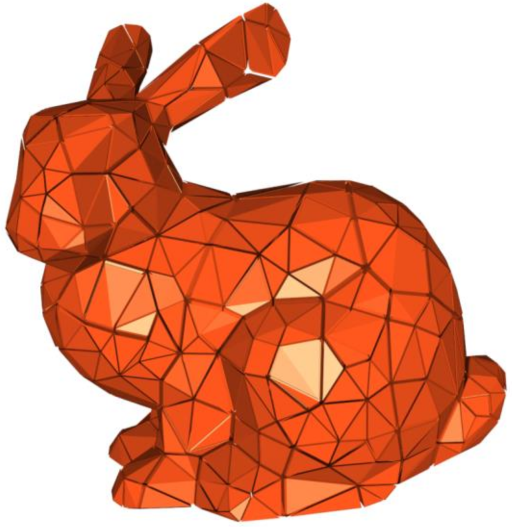


vulcanized natural rubber

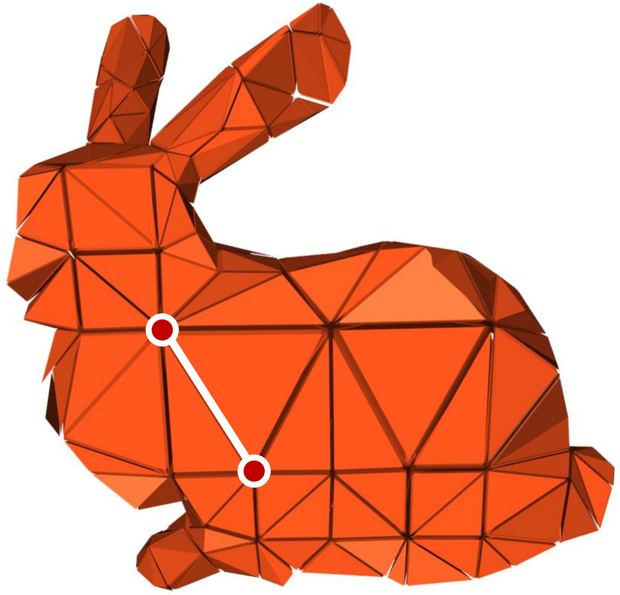
Simulation

Approach

- Create tetrahedral mesh
  (Delaunay tetrahedralization, upcoming tutorial)



- One particle per vertex
- One distance constraint per edge
- One volume constraint per tetrahedron



PBD Algorithm (recap)

$\Delta t_s \leftarrow \Delta t / n$
while simulating
  for n substeps
    for all particles i
      $\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t_s \mathbf{g}$
      $\mathbf{p}_i \leftarrow \mathbf{x}_i$
      $\mathbf{x}_i \leftarrow \mathbf{x}_i + \Delta t_s \mathbf{v}_i$
    for all constraints C
      solve$(C, \Delta t_s)$
    for all particles i
      $\mathbf{v}_i \leftarrow (\mathbf{x}_i - \mathbf{p}_i) / \Delta t_s$

solve$(C, \Delta t)$:

for all particles i of C
  compute $\Delta \mathbf{x}_i$
  $\mathbf{x}_i \leftarrow \mathbf{x}_i + \Delta \mathbf{x}_i$

Solving a General Constraint (recap)

Compute the scalar value λ (same for all participating particles):

$$\lambda = \frac{-C}{w_1|\nabla C_1|^2 + w_2|\nabla C_2|^2 + \cdots + w_n|\nabla C_n|^2 + \frac{\alpha}{\Delta t^2}}$$

Compute correction for point $\mathbf{x}_i$ as: $\Delta \mathbf{x}_i = \lambda w_i \nabla C_i$

C Constraint function, zero if the constraint is satisfied

∇C_i Gradien to f C, how to move $\mathbf{x}_i$ for a maximal increase of C

w_i Inverse mass of particle i

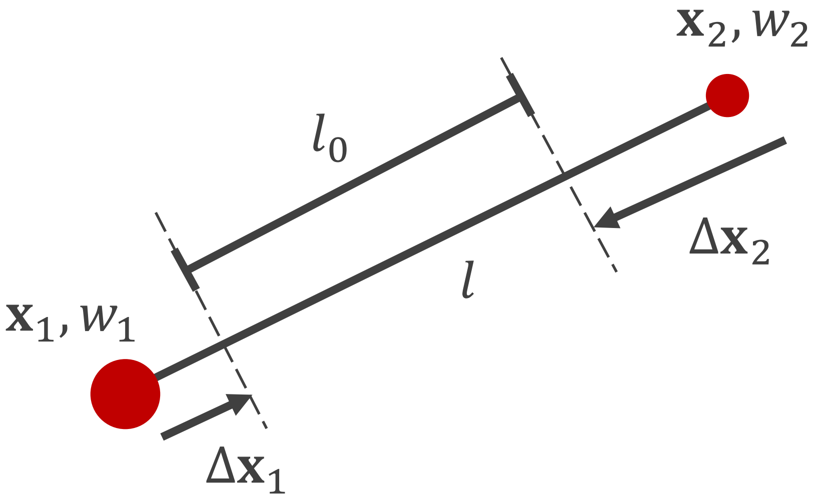
α Inverse of physical stiffness, stable for infinite stiffness ($\alpha = 0$)!

Distance Constraint

$$C = l - l_0$$

$$\nabla C_1 = \frac{\mathbf{x}_2 - \mathbf{x}_1}{|\mathbf{x}_2 - \mathbf{x}_1|}$$

$$\nabla C_2 = -\frac{\mathbf{x}_2 - \mathbf{x}_1}{|\mathbf{x}_2 - \mathbf{x}_1|}$$



Volume Conservation Constraint

$$C = 6(V - V_{\text{rest}})$$

$$V = \frac{1}{6}\left((\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1)\right) \cdot (\mathbf{x}_4 - \mathbf{x}_1)$$

$$\nabla_1 \mathrm{C} = (\mathbf{x}_4 - \mathbf{x}_2) \times (\mathbf{x}_3 - \mathbf{x}_2)$$
$$\nabla_2 \mathrm{C} = (\mathbf{x}_3 - \mathbf{x}_1) \times (\mathbf{x}_4 - \mathbf{x}_1)$$
$$\nabla_3 \mathrm{C} = (\mathbf{x}_4 - \mathbf{x}_1) \times (\mathbf{x}_2 - \mathbf{x}_1)$$
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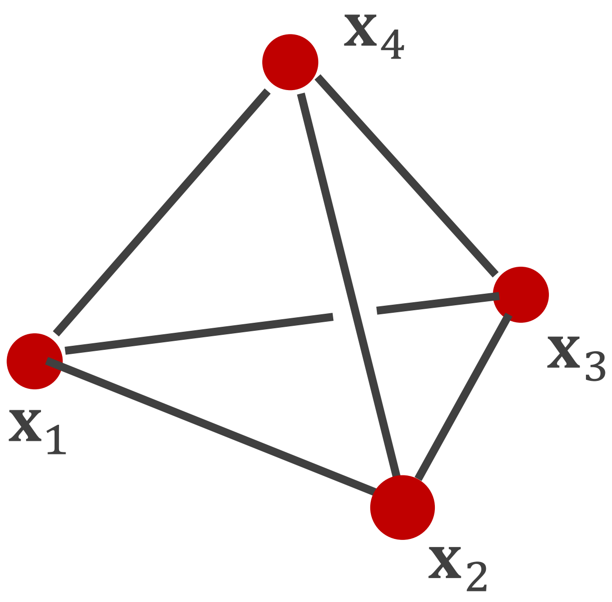

Let’s do it…